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ABSTRACT

Teacher questions are an important part of a student-inquiry classroom. This research examines two different student-centered settings to determine the teacher questions that engaged students in mathematical thinking. It reports on questions asked in both a research setting and a high school classroom. Discursive and retracing questions are defined as asking a student to contribute to an ongoing discourse and consider an old idea, respectively. These questions started strands of student engagement in mathematical thinking. Confirmation, justification, and clarification questions were also asked by both teachers and kept students engaged in mathematical thinking. (Author)

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QUESTIONS THAT ENGAGE STUDENTS IN MATHEMATICAL THINKING

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Teacher questions are an important part of a student-inquiry classroom. This research examines two different student-centered settings to determine the teacher questions that engaged students in mathematical thinking. It reports on questions asked in both a research setting and a high school classroom. Discoursive and retracing questions are defined as asking a student to contribute to an ongoing discourse and consider an old idea, respectively. These questions started strands of student engagement in mathematical thinking. Confirmation, justification, and clarification questions were also asked by both teachers and kept students engaged in mathematical thinking.

Recent reform efforts in mathematics education promote communication as an essential part of the mathematics classroom. Communication makes ideas objects of reflection, discussion, and refinement as part of the process of organizing, consolidating and giving meaning to these ideas (NCTM, 2000). If students are expected to explain their ideas, a question arises about what influence, if any, a teacher's response to those explanations has on student thinking. While a range of pedagogical responses is possible, a key way to facilitate communication and discover student thinking is teacher questions.

If mathematics teachers teach in an environment filled with conversation and where student ideas are valued, then teachers must know what questions engage students in mathematical thinking. While there have been many studies published on teacher questioning, most of them focus on the teacher. Some of this general education literature examines the effectiveness of teaching (Gall & Rhody, 1987) and categorizes teacher questioning (Cotton, 1989; Cunningham, 1987) according to student achievement. Other literature looks at how questioning strategies can involve more students in the learning process by provoking thoughtful responses (Wigle, 1999; Mewborn & Huberty, 1999). While bodies of mathematics education literature examine teacher questioning of students (Maher & Davis, 1990; Maher, Davis & Alston, 1992; Maher & Martino, 1992; Martino & Maher, 1999; Martino & Maher, 1994; Vacc, 1993) and students' learning and understanding of mathematics (Cobb, Wood & Yackel, 1992; Davis & Maher, 1990; Yackel & Cobb, 1996), there has not been an integration of these two research areas. Influenced by the call for communication in student-centered classrooms, the purpose of questioning is to help students explore their ideas during the communication process. Therefore, teachers need guidelines for questions that engage students in mathematical thinking. During a student and teacher conversation, teachers cannot use prepared questions or prescribed strategies. Teacher questions need to be based on the responses received from the student so the

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teacher can continue the conversation in order to engage the student in mathematical thinking.

The purpose of this study is to address this imbalance in the literature and inform mathematics education practice by classifying and describing teacher questions that engage students in mathematical thinking within a student-centered setting. The following research questions frame the study: 1) What kinds of questions do mathematics teachers in two different settings ask?; 2) What questions engage students in mathematical thinking?

Theoretical Framework

The call for communication is influenced by research in constructivism and discourse. These two research areas lead to the promotion of classrooms where students take an active role in learning and gain mathematical knowledge through social interaction and experience (Noddings, 1990; von Glasersfeld, 1990). As a result, teacher questions are an important aspect of student engagement in mathematical thinking. Since this study focuses on teacher questions, literature in this area supports the framework for the study.

Teacher Questioning and Questions

Questions are a valuable teaching tool and by far the most used technique of teaching (Clegg, 1987). Questions serve many purposes, such as initiating discussion and reviewing material, but the purpose of the question determines the kind of question asked by the teacher (Cunningham, 1987). Research, from general education classes, on questions provides many classification schemes, which label questions according to a particular cognitive level of student thinking.

Woolfolk (1998) suggests categorizing questions into divergent questions, which have many possible answers, or convergent questions, which have one right answer. Cotton (1989) found the majority of researchers conducted similar dualistic comparisons about questioning and Cunningham (1987) provides a more extensive list of questions for teachers to ask based upon the cognitive level of student responses.

Another view of questions is to categorize them in a hierarchy. The most widely used hierarchy is Bloom's taxonomy, where questions are labeled from simple to complex cognitive objectives (Woolfolk, 1998). Wolf (1987) suggests a different hierarchy, which focuses solely on what he considers challenging questions, from observations in his classroom.

A third view of questions is their role in effective teaching. This more current research shows that communication and questioning are part of a larger equation for effective teaching (Glenn, 2001). Effective teachers tend to ask more process questions, asking for explanations, though the majority of questions were product questions, asking for a single response (Reynolds & Muijs, 1999).

Questioning in Mathematics

Research regarding questioning in mathematics classrooms also focuses on classification schemes. Hiebert and Wearne (1993) identify four types of questions: recall, describe strategy, generate problem, examine underlying features. Vacc (1993) cites three categories of questions that occur in the classroom; factual, reasoning, and open based on a study by Barnes (as cited in Vacc, 1993) on questioning in classroom instruction. Vacc (1993) concludes that teachers asking factual questions will find out the specific facts their students know, but teachers who ask questions in the open category gain information about their students' cognitions. While the conclusions are helpful, there are no specific questioning guidelines offered or connections to student mathematical thinking.

Additional research examines questioning strategies and how questions can get students to communicate mathematical ideas. Mewborn and Huberty (1999) advocate a question-listen-question strategy in order to encourage discourse. The study reports how teachers, using questions from the NCTM Standards, improved discourse, but does not provide a classification for the questions in the study.

An examination of teacher questioning, by Martino and Maher (1999), explains how the timing of questions can determine a student's understanding of a mathematical idea after students construct their own ideas. In order to build upon student ideas, the research proposes asking students questions that lead them to justifying their ideas. Dann, Pantozzi and Steencken (1995) also examine teacher and student discourse and recommend that teacher's ask questions, which promote student interaction to help extend their ideas and justify their conclusions. Research in this area supports the idea that questions can encourage students to talk about mathematics, but this research does not classify questions, which promote mathematical thinking.

Research Design

This case study examines two settings in order to provide insight into questions teachers ask and what questions engage students in mathematical thinking.

Participants and Settings

The first setting for data collection is an urban high school classroom. Thirty-five honors students, comprised of sophomores and juniors, work on calculus problems at individual desks in rows. The teacher being observed is a 30-year veteran of the school system and holds a doctorate in education.

The second setting is a component of a longitudinal study on the development of proof making in students[1]. Eighteen high school students, entering their fourth year, work in groups on an open-ended precalculus mathematics problem in a library of a high school. The students are seated in groups and five teacher/researchers interact with the students. Six students, sitting at the same table (two males and four females), and one teacher/researcher, an experienced professor of mathematics and mathematics education at the university level, have been selected for this study.

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Data Collection and Analysis

Eighty-minute videotape observations of the high school classroom sessions and two-hour videotape sessions of the two-week institute, both from a consecutive three-day period, comprise the data for this study. Field notes from both settings account for events beyond the view of the camera and summarize the class activity.

The analysis of the data for this study follows the model provided by Powell, Francisco & Maher (2001). Videotape were digitized onto CDs and each CD was summarized. Repeated viewing of the data allowed for the identification of critical events. A critical event, for this study, is defined as a teacher-student interaction where student mathematical thinking could be followed. The codes emerging from the data are: T(r): Teacher asks a student to consider an old idea; T(d): Teacher asks a student to contribute to the ongoing discourse; T(c): Teacher asks the student to clarify their statements or ideas; T(j): Teacher asks the student to justify their statements or ideas; T(con): Teacher confirms the student and teacher both agree on what has been done or said; and T(f): Teacher follows the student's idea or suggestion. It is possible for one question to receive multiple codes.

Results

The episodes selected demonstrate strands of student engagement in mathematical thinking. The strands presented here show teacher questions, with codes, that were common in both settings.

Summer Institute

The task the students are working on during the Institute is called Placenticeras, developed by Bob Speiser. The first part of the task is to draw a ray from the center of the shell in any direction. Then using polar coordinates as a way to describe the spiral of the shell, the students are to make a table of r as a function of theta. The question for the task is what could be said about r as a function of theta.

Episode 1

During the morning session, the students make measurements from a picture of the shell and create a table. They talk with each other about how to enter the points from their table into the TI-89 calculators in order to graph a function. Alice interacts with the group throughout the morning session about the where their measurements come from and what the number representing an average means. The students tell her their measurements go from the origin out to the shell, a radius, and the average is the distance traveled along the shell per 90 degrees. The students develop the equation r equals .039337 times theta, where .039337 is a result of their work on determining an average. In this day's afternoon session, Alice returns to the group and questions Victor.

Alice	T(r)	If we're saying, you told me the radius, which is he distance out to an point, is equal to that number that you came up with which was .039337
Victor		mm, hmm.
Alice	T(con)	Is that right?
Victor		I guess that, that's right for what I said, not right, like correct.
Alice		Oh, no. I'm, I'm just saying.
Victor		Okay.
Alice	T(con)	That's what you got and that's what you, that's what you based your spiral on.
Victor		mm, hmm.
Alice		Times theta.
Victor		Right.
Alice	T(c)	And theta for this number was in degrees or in radians?
Victor		Degrees.
Alice	T(j)	Okay, and so then shouldn't you be able to plug in a number of degrees and get one of your points on the spiral?
Victor		Say that again now.
Alice	T(r, c)	If this is an equation that says r is equal to .039337 times theta, what happens if you plug in a number of degrees? And multiply it by .039337. What should you get?
Victor		A number of radius I guess. The number of the, the radius, right?
Alice	T(con)	The, the length of a radius? Is that what you should get?
Victor		Uh huh.
Alice	T(con)	But now, isn't that what these things are?[points to values on their spiral]
Victor		Right.
Alice	T(r, j)	Well shouldn't she be able to check by going backwards or not?

Victor shows some frustration with Alice, but agrees to check out if this idea works.

Alice	T(c)	What's the degree to get this guy out here[points to the last value on their spiral]?

Victor	Uh, 6.55 radians. Which is about, 6.5 um, 6.5 times 180. 1107.
Alice T(con)	Which is almost your total, wasn't it?
Victor	.0, hmm, .0 what, three.
Alice	9337.
Victor	No, you don't get the radius. You don't get the radius. Oh dang.
Alice	No, I just.
Victor	You don't get the radius, you get the length.
Alice T(con,c)	What do you get? do you get the length?
Angela	Is it length from like, length like this, that's what you get?
Victor	Yeah.
Alice T(c)	So is that what you are getting?
Victor	That's what I mean. [Referring to the term length rather than radius.]

Alice asks the students to check two more values with their equation.

Alice T(c)	So, so what, what is the thing over here? What is this thing over here? I mean what is it, what's the name of it?
Victor	I don't know. um, (inaudible) of the spiral.
Angela	Some kind of arc length type deal. I don't know.
Alice	T(con) Well arc, arc, whatever. Arc length, Isn't that what you call that?
Victor	Arc something.

Episode 2

During the last day of working on this task, the students are talking about what they are going to present to the rest of the groups at the summer institute. Alice asks Robert about the two equations the group developed to model the growth of shell. Robert explains how the equations came from the scatter plot the group created, which was made from their table. He also relates the *x* and y values represented on their scatter plot to the values of *r* and theta measured on the picture of the shell. After finishing his explanation, Alice asks Angela about what Robert said.

Alice T(d)	Angela does that make sense to you at all? or not?
Angela	Sort of. I like get lost with all this stuff. I hate this.
Alice T(f,c)	What, what are the, what is the sort of question that throws you?

Angela	Like, I get like little bits and pieces of what he's explaining, but I don't really get all of it.
Sherly	What don't you understand?
Angela	I don't know.
Alice T(c)	I think I hear (inaudible), she doesn't even know what she's asking. Um what, what are you asking...in this?
Angela	I don't have like a specific question. I just don't like understand the whole, like everything you just explained. Like why that, the whole thing, like what you were saying like why it's like the spiral unraveled or something like that. Like I don't, you explained the points and just trying to follow and just didn't.
Robert	It's kind of hard to explain.

The students ask each other if they understand what Robert explained.

Alice T(d)	Could you, could you try Michelle, to explain. Cause every time one of you explains it, it helps me a little, little more. This is really just as foggy for me, Angela, as it is for you. I am even further away than you, from this stuff, because I don't understand the calculator either. So Michelle could you try it again.
Michelle	Okay, um. Ooh. Alright if you took, let me draw a piece, the spiral, and you picked like certain points, whatever ones they were. Right? Robert?
Robert	Yeah.
Michelle	Okay (laugh), just checking. And like um, you're doing the ninety degree intervals, which is like the pi/2, you know what I mean, and like you graph them...Okay, then you're saying that r would be the radians?

Michelle, with the help of Robert, continues to discuss with Alice the meaning of the values the group measured from the picture of the shell. Alice leaves the discussion after asking the group to show how they used a rubber band to get the lengths in their table.

High School Classroom[2]

The task the students are working during these observations is to find the area under a curve over a given interval. The teacher provides the students with a function and then calls students to the board to work on a specific part of the task. For episode one, which occurs on the second day of observation, the function given is $f(x) = 2x^3 + x^2$ over the interval [1,3].

Episode 1

The students take the derivative of $f(x) = 2x^3 + x^2$ to get $f(x) = 6x^2 + 2x$. Using the properties of its first and second derivatives, the class draws a graph for f(x). The students find the area under the curve using four rectangles. The teacher asks the students to express the area using sigma notation and then assigns various students in the class a number of rectangles to use to calculate the area with a TI-89 calculator. After the class sees the area is approaching 60 as the number of rectangles increases, the teacher asks a student to write the area using limit notation. The student, Cedric, writes $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}$.

DrG		Question Keri.
Keri		Shouldn't it be 2 over *n* cause that is how you did everything else.
Cedric		(inaudible) [calls on a student to answer]
Student		It should be. That first expression should be *n* over 2, which that would be. So that top number which is *n* is four. So the number in the denominator is four.
Keri		But that.
Student		What?
DrG	T(r, f)	Keri, let us hear your argument again please.
Keri		Okay, it should be 2 over *n*. Because like to get one half, you do two over four. Cause it was like you have four triangles and the area two. Two. So it was two over four, which is one half and you had two over ten which is one fifth. And so on. So wouldn't it be two over *n*.
DrG	T(con,d)	Any disagreement? Everyone understand that. What'd she say Chuck?
Chuck		She said that since those four triangles and those were split into two parts. I mean four rectangles. It was two over *n*, which was reduced to one half and that's how we got one half.
DrG	T(con)	Is that what you said Keri?
Keri		Uh, not really.
DrG	T(r,c)	Try it again, Keri. And then we'll back it up and try and have him run it through again.
Keri		Okay, I don't know how. I'm sure how to explain it. Yeah, You had

okay. You had like two, one to three, three minus one is two. So, you have like two spots and you're doing it for four rectangles. So you did two over four is equal to one half and then you did two over ten is one fifth and so on. So to get it for n, you get two over n. I think that's what I said.

DrG T(d,con) So, now you try and say it Chuck.

Chuck Yeah, fine. Uh, Since we're going from one to three, the top part is two. And since there's four rectangles, the bottom part is four.

DrG It's the length of the interval divided by the number of rectangles. [points to the expression written on the board]

The student finishes the expression for the area and writes "= 60". The class uses the calculator to graph the derivative function and find the area under the curve for the interval [1,3]. The teacher draws the students' attention to the integral notation on the calculator and introduces paper and pencil integral language using formal, graphical and $f(x)$ notation. The teacher also explains how to use the TI-89 to calculate the integral of their given derivative function. After this explanation, a new problem to find the area under the curve $f(x) = 2x^2 + 3x$ over [0,1] is introduced, and a student is called to the board to calculate the parent function. After calculating the parent function by reversing the power rule for derivatives and finding the area using this method, the teacher asks another student to set up the limit equation, which they will determine for homework.

Marc So just, uh, write the equation. [Student writes $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}$]

DrG T(r,j) Now, why one over n? Awhile ago it was two over n.

Marc Because the uh, zero to one, not one to three.

DrG Oh, okay.

Conclusions

The two teachers asked many questions during their interactions with students. However, two questions started student engagement in mathematical thinking, while other questions kept students engaged in mathematical thinking. Retracing and discoursive questions, coded T(r) and T(d) respectively, started the strands of student thinking in the episodes selected. The additional questions of clarification, T(c), justification, T(j), and confirmation, T(con), allowed the teachers to keep the student(s) engaged in mathematical thinking.

Both teachers used these questions to engage students in mathematical thinking, but each teacher asked questions for various reasons and purposes. Alice would ask

a question after listening to the students' conversation. Dr. G would ask questions with a goal in mind for the students. His initial questions did not always come from something the students said.

Each teacher utilized retracing questions for different purposes. In episode one of the summer institute observations, Alice questions students about what their measurements mean so they could revisit their own ideas. She continually returns to this idea throughout the day to get the students to clarify their thinking. Finally, in the afternoon session, Victor realizes it is not the radius he is talking about, but rather the arc length of the spiral. Retracing questions allowed Victor to revisit his mathematical thinking about this topic until he clarified the meaning of his measurements.

In the high school setting, the teacher asks retracing questions to emphasize an earlier idea to the entire class. Keri indicates the width of interval should be two over *n* and justifies her reasoning. The teacher returns to this idea when a different student is writing the value of the width of the interval for a new problem to emphasize to the class what Keri stated earlier.

Both teachers ask discoursive questions to check for other students' understanding of one student's idea. In episode two of the summer institute, Alice asks Angela if she understands what Robert just explained. Since Angela states she does not understand, Alice asks Michelle to see if she understands and can explain Robert's ideas about the group's work. Michelle becomes engaged in mathematical thinking and, with the help of Robert, restates the group's ideas. In the high school classroom, the teacher mimics Alice's objective. He has another student explain what Keri said about the interval width for the rectangles in order to check for understanding of other class members.

Retracing and discoursive were the two main types of questions the teachers in both settings used to engage students in mathematical thinking. The transcripts presented also show both teachers using many clarification, justification, and confirmation questions to extend student mathematical thinking. Even though both teachers used these types of questions, Alice asked more clarification and confirmation questions, while Dr. G asked all three kinds of questions. However, the episodes presented only show a very small sample of the overall student-teacher interactions from the two settings over a short observation period. Further quantitative analysis of the entire transcript may provide further insight into the quantity of types of follow-up questions asked by each teacher.

Studies about questioning in the mathematics classroom call upon teachers to ask initial questions that provoke thoughtful responses from students, but to follow initial questions with others that help students clarify, justify and extend their thinking (Dann, Pantozzi & Steencken, 1995; Mewborn & Huberty, 1999). This research shows that teachers can use discoursive and retracing questions as initial questions to engage students in mathematical thinking. Teachers can follow these initial questions with clarification, justification, and confirmation questions to extend and continue student mathematical thinking.

Notes

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[2]Names of students in this setting have been changed.

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